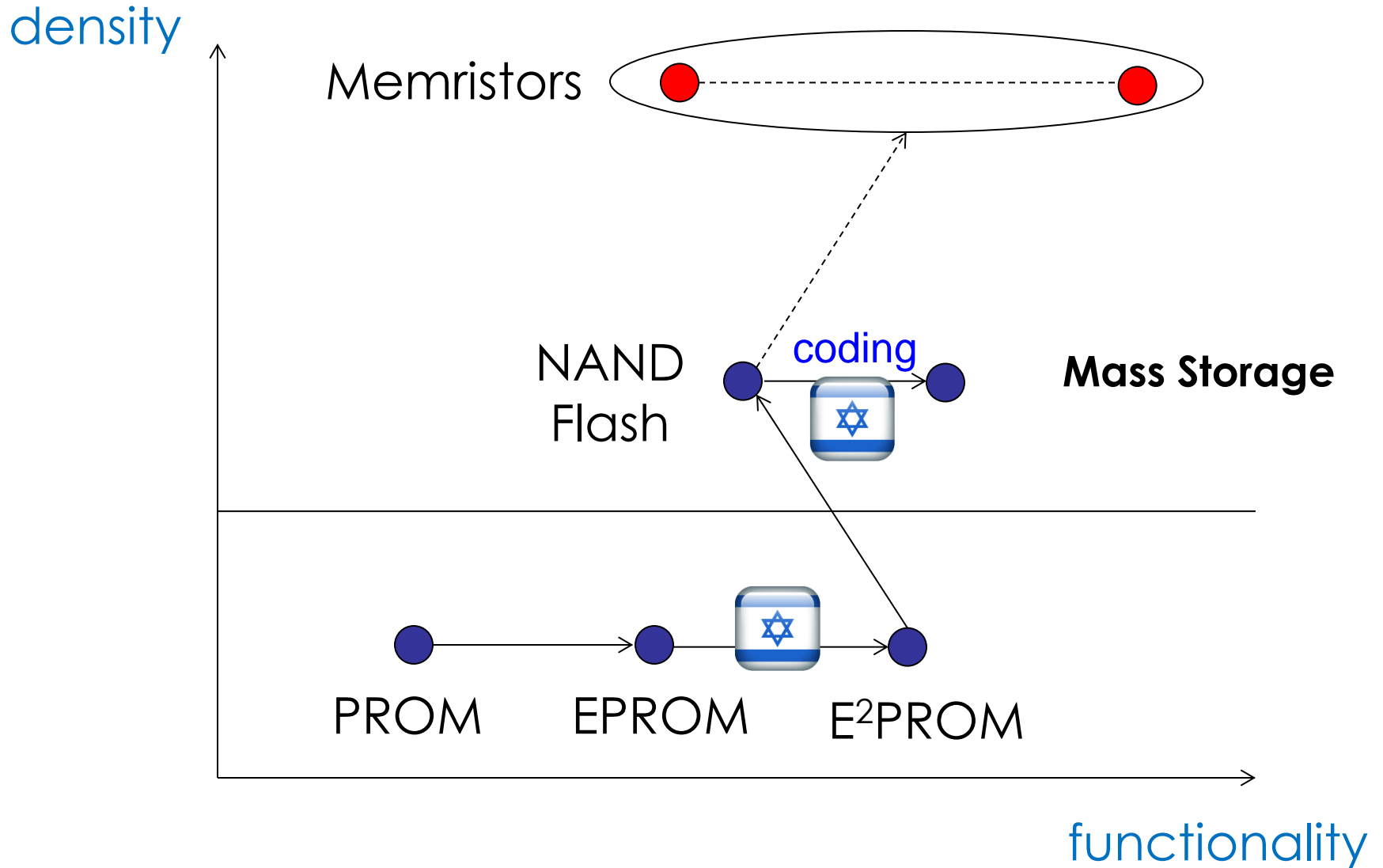




Seminar on Coding for Non-Volatile Memories
236803/048704 – CS/EE Departments, Technion

MEMRISTORS / RESISTIVE MEMORIES

Non-Volatile Memory Progression

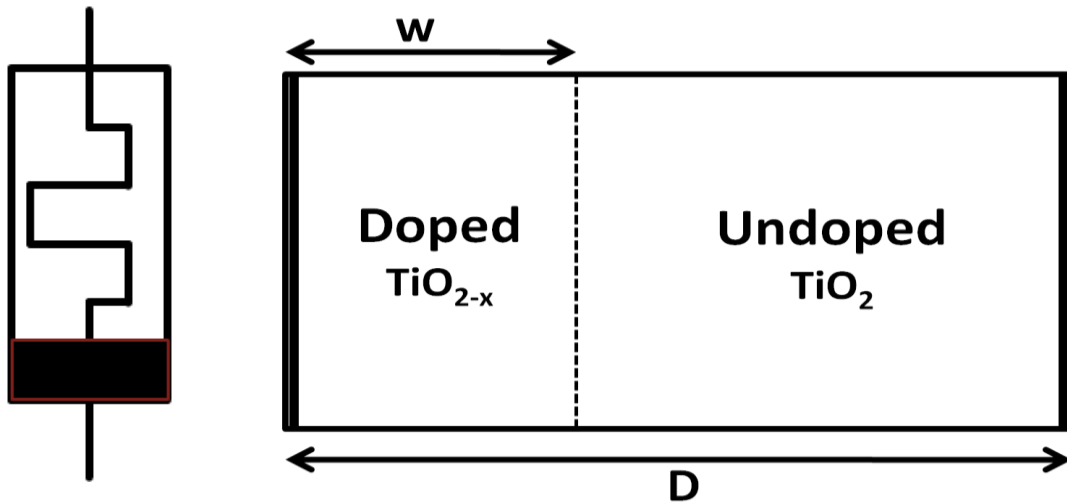


Outline

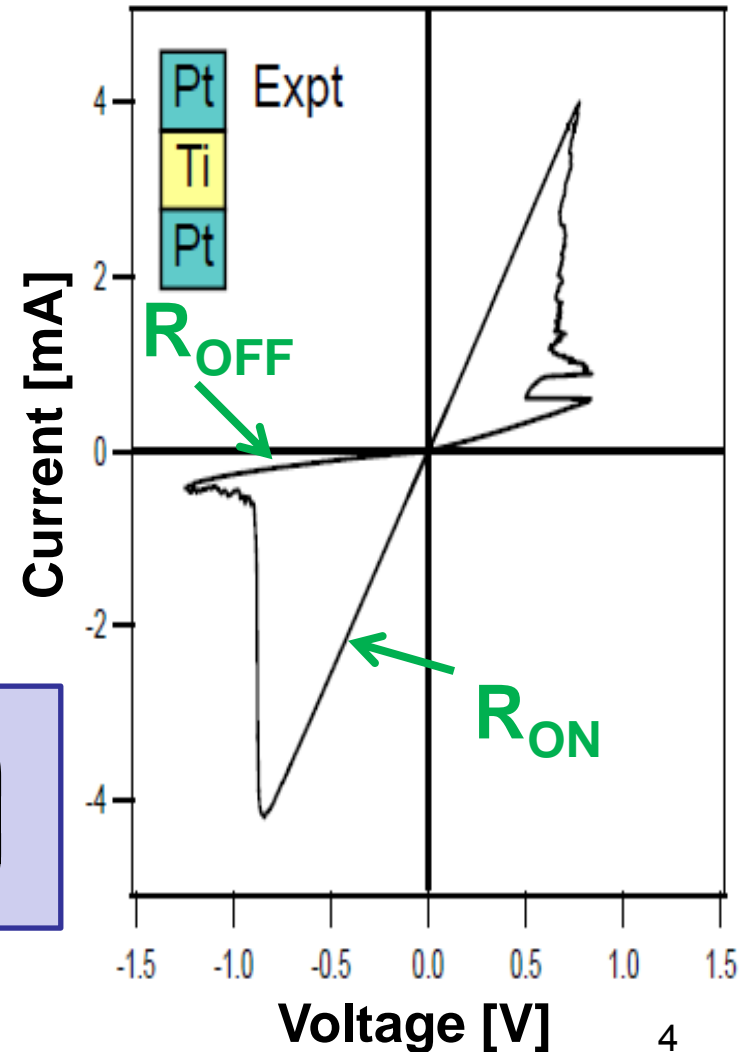
- Memristors
 - Promising storage technology
- Sneak Paths
 - Main functional limitation
- Coding for Sneak-Path mitigation
 - Sneak path elimination
 - Sneak path as random error source

Memristors

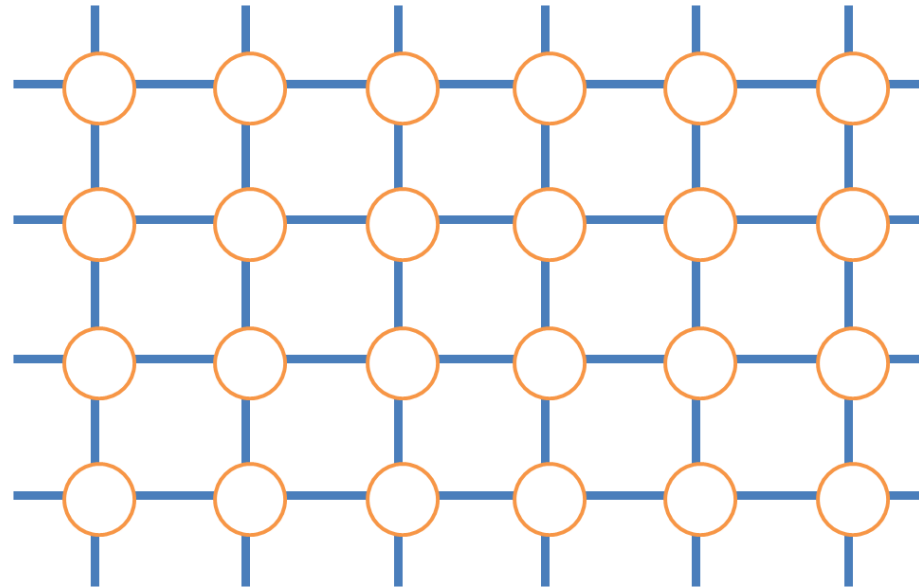
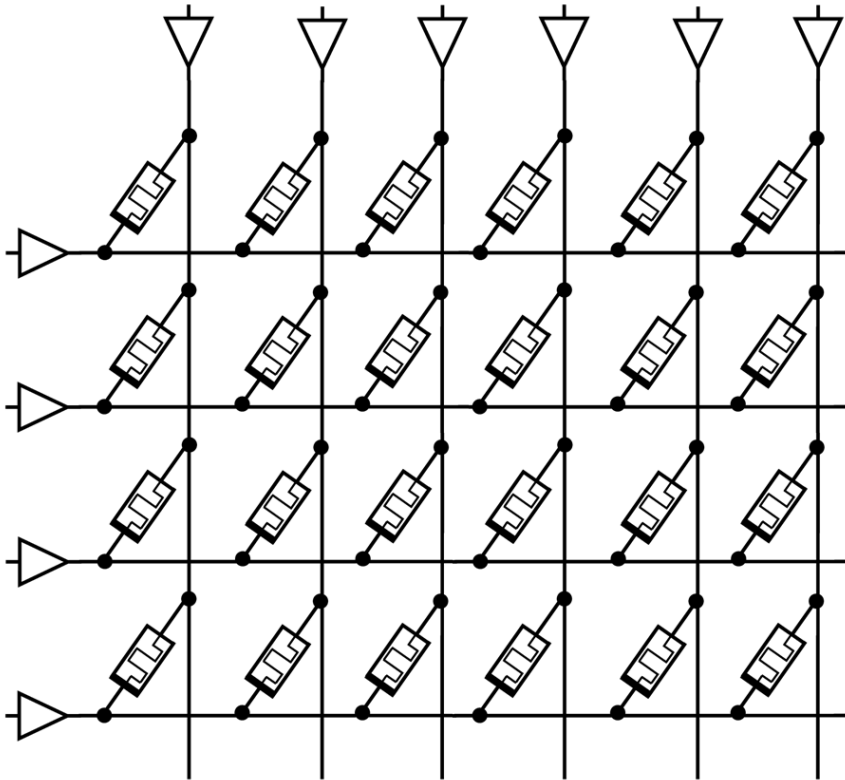
- 2008 Hewlett Packard

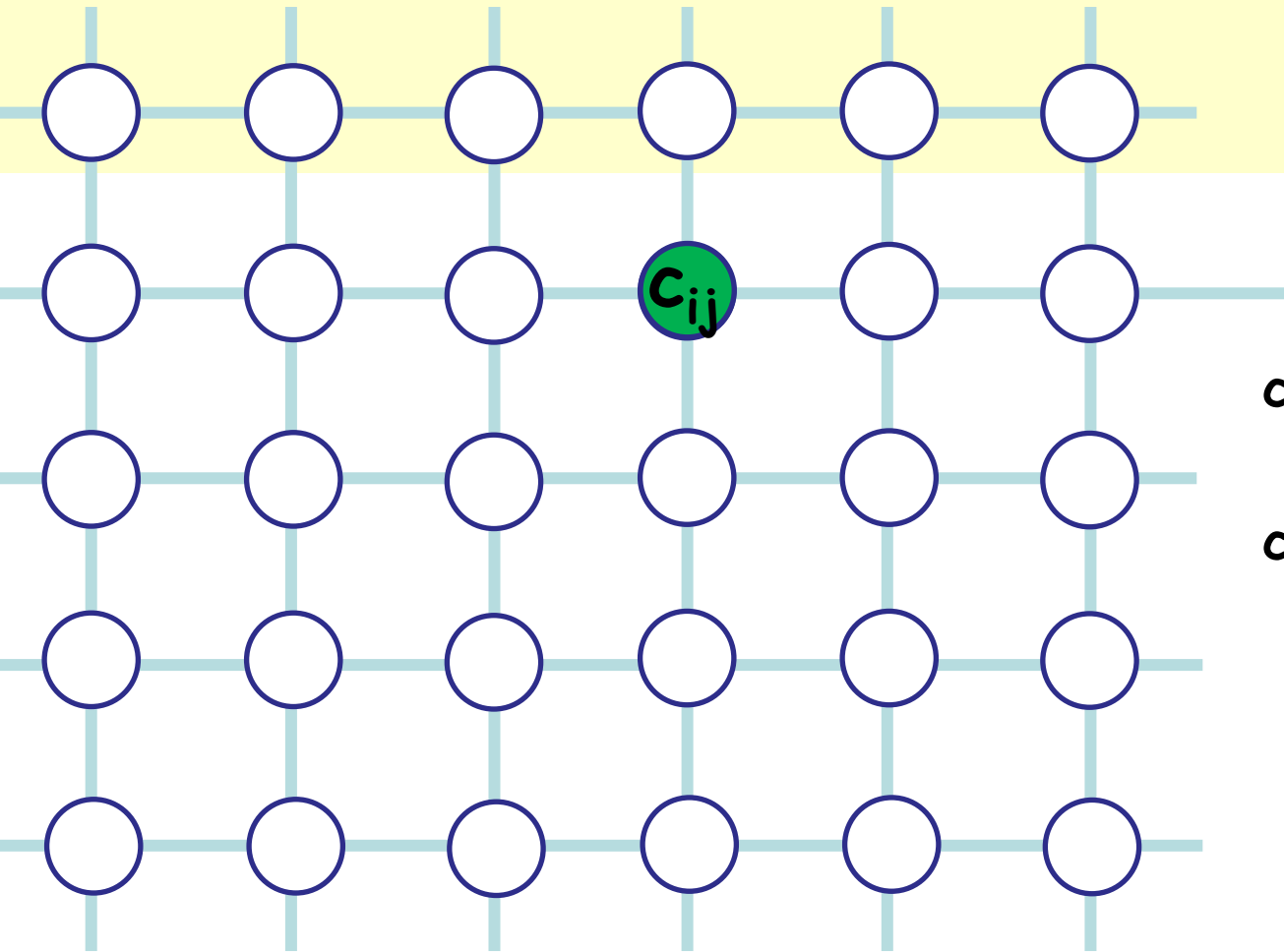


$$M(q) = R_{OFF} \left(1 - \frac{\mu_v R_{ON}}{D^2} q(t) \right)$$



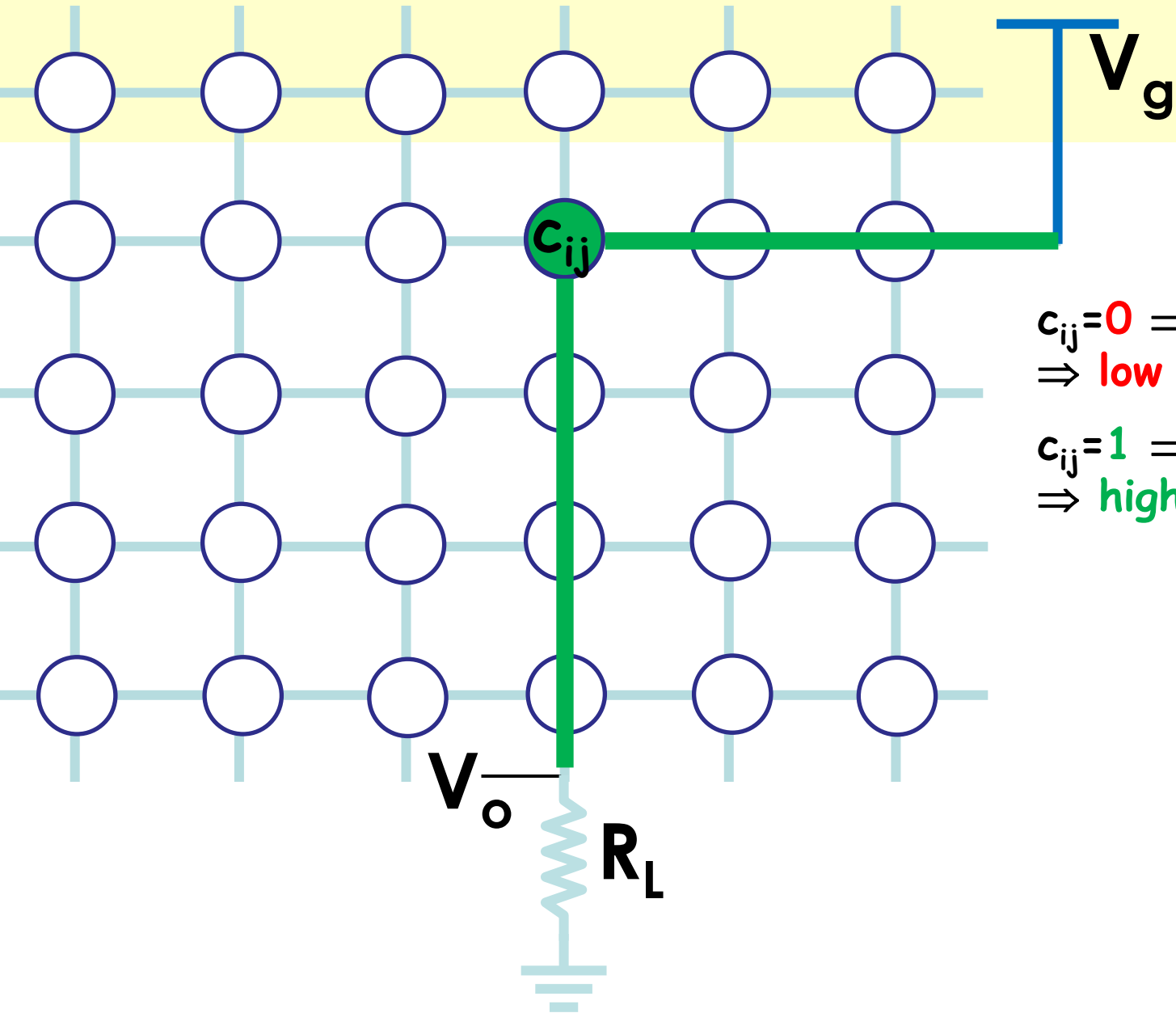
Crossbar Arrays





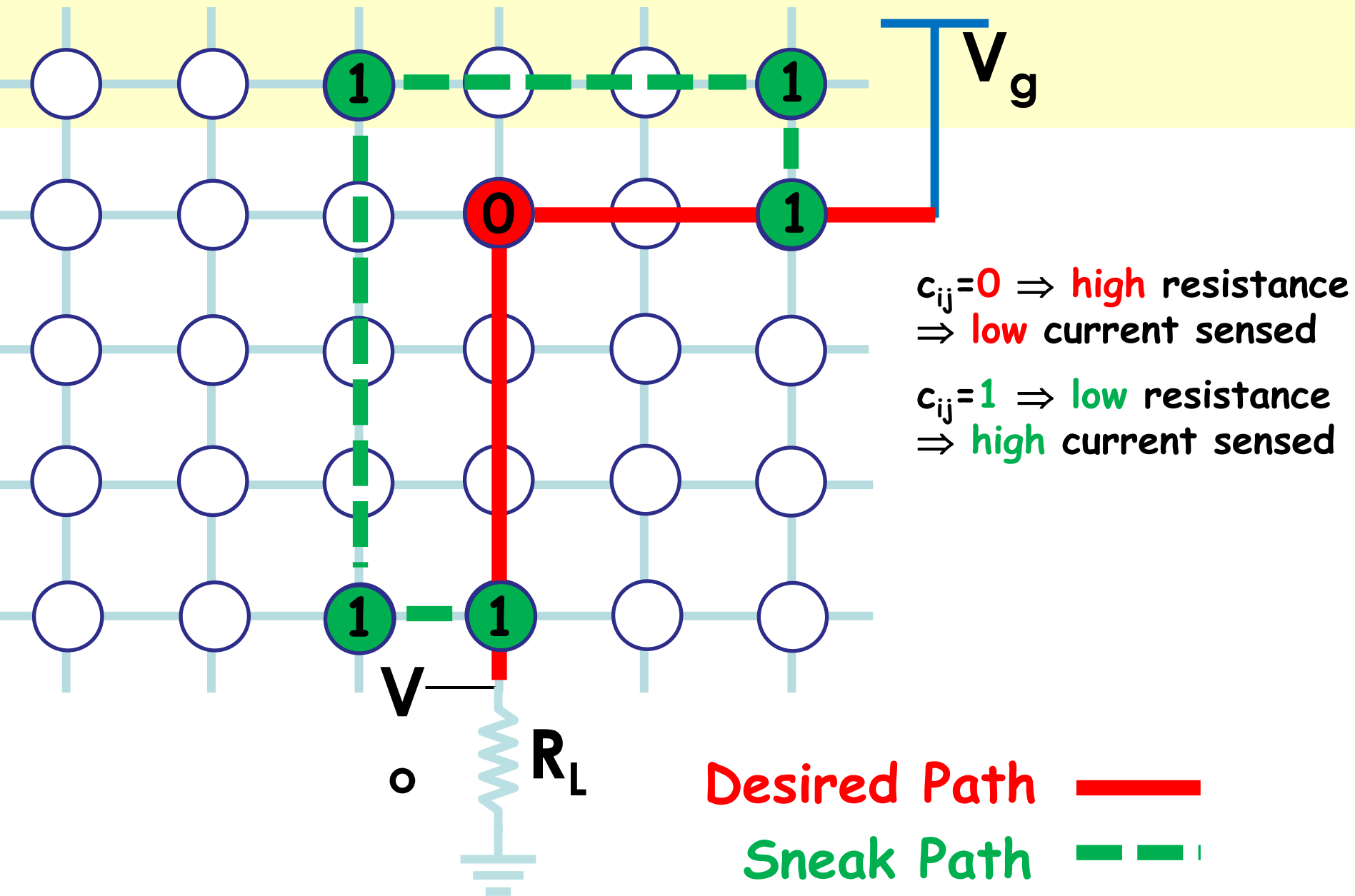
$c_{ij}=0 \Rightarrow$ **high** resistance

$c_{ij}=1 \Rightarrow$ **low** resistance



$c_{ij}=0 \Rightarrow$ **high** resistance
 \Rightarrow **low** current sensed

$c_{ij}=1 \Rightarrow$ **low** resistance
 \Rightarrow **high** current sensed



Part 1

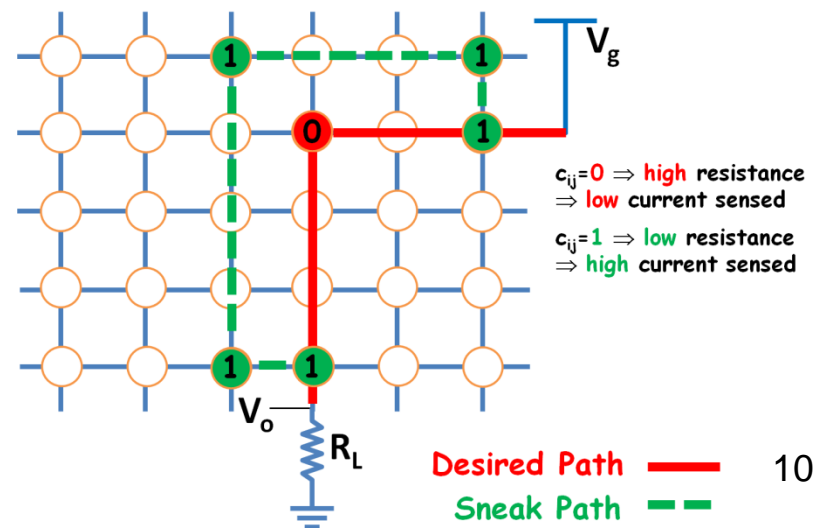
Sneak path **elimination**

Sneak Path

- An array **A** has a **sneak path** of length $2k+1$ affecting the (i,j) cell if
 - $a_{ij}=0$
 - There exist r_1, \dots, r_k and c_1, \dots, c_k such that

$$a_{ic_1} = a_{r_1c_1} = a_{r_1c_2} = \dots = a_{r_kc_k} = a_{r_kj} = 1$$
- An array **A** satisfies the **sneak-path constraint** if it has no sneak paths and then is called a **sneak-path free array**

		1			1
			0		1
		1	1		



Characterization of Sneak Paths

- **Theorem:** An array **A** has a **sneak path** if and only if it has an **isolated zero-rectangle**

	1			1	
	0			1	

- **Proof:**

		1			1
		0	0		1
		1	1		

		1			1
		1	0		1
		1	1		

Row/Column Overlap Condition

- Lemma:

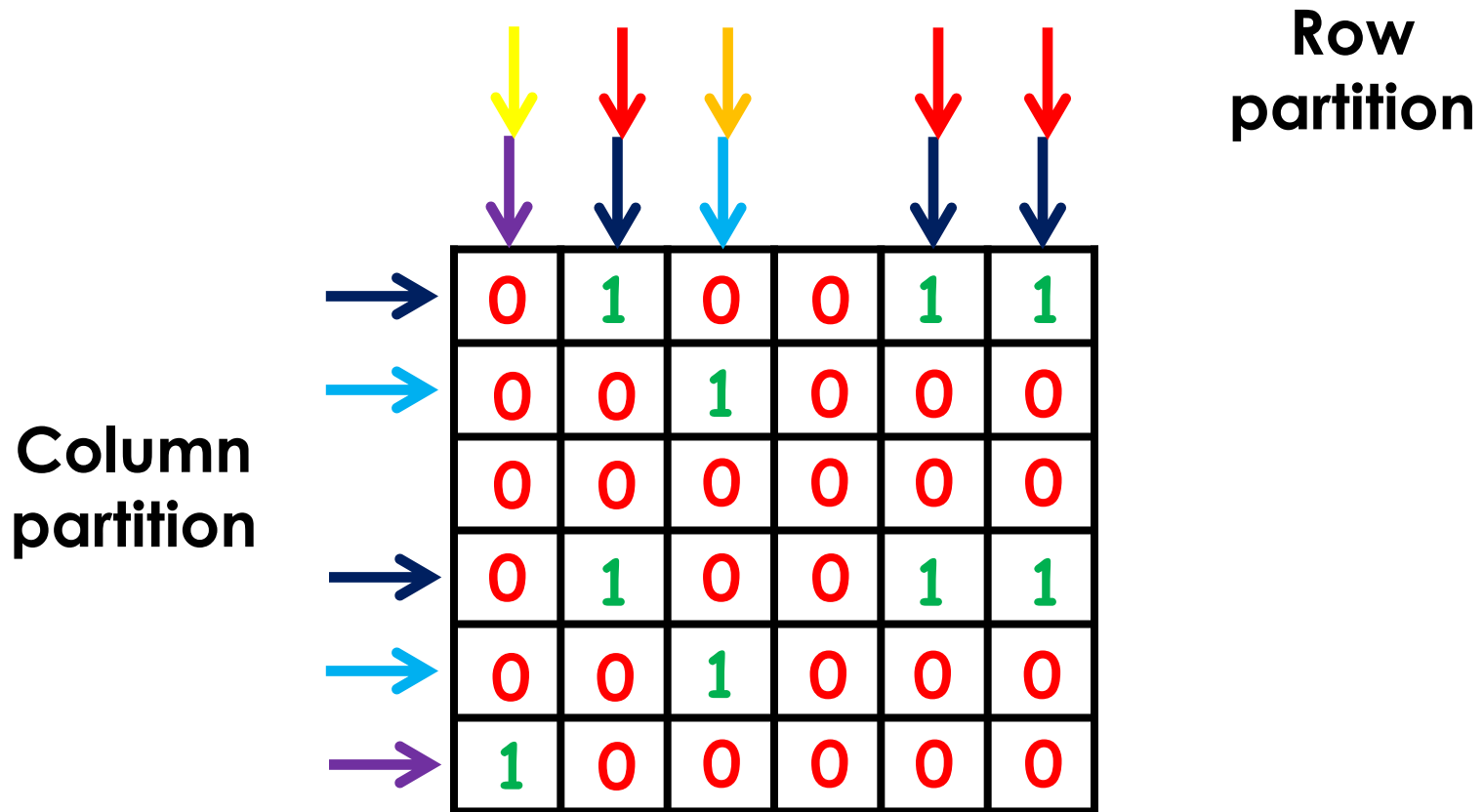
An array has **no isolated zero-rectangles** iff the 1s in every two rows/columns either **completely overlap** or **are disjoint**.

Disjoint

0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
1	0	0	0	0	0

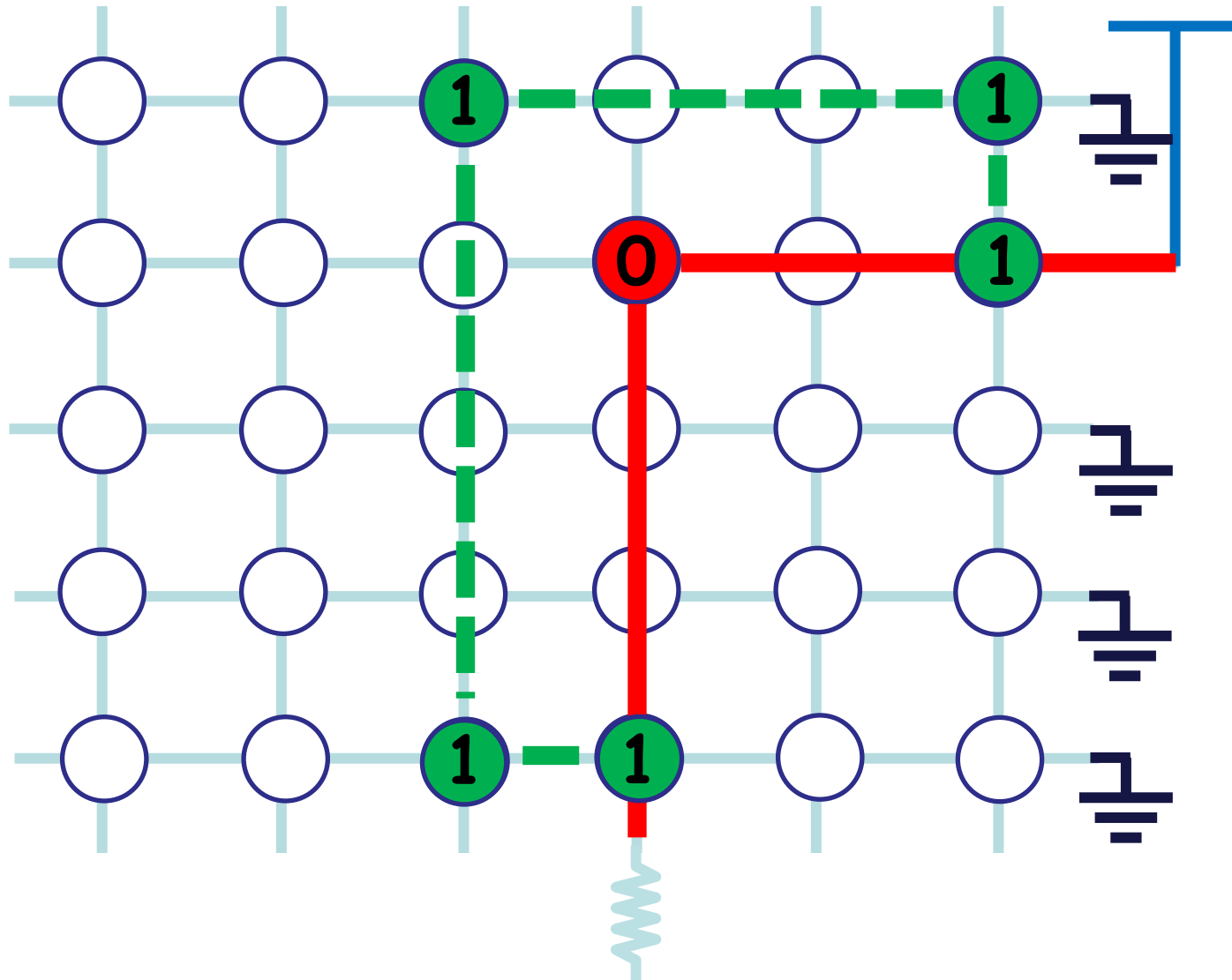
**Complete
overlap**

Encoding Sneak-Path Free Arrays

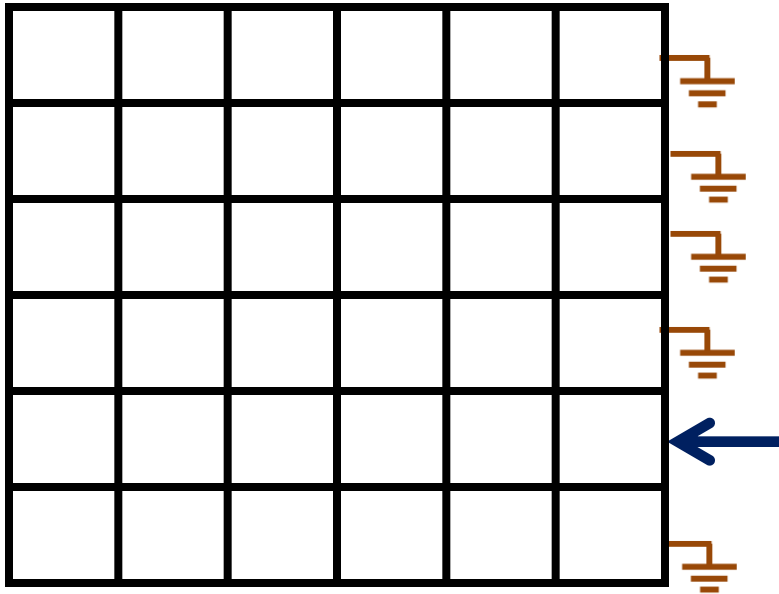


$\approx 2n \log n$ arrays, when $n=m$

Grounding – an EE Solution



EE or IT?



0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
1	0	0	0	0	0

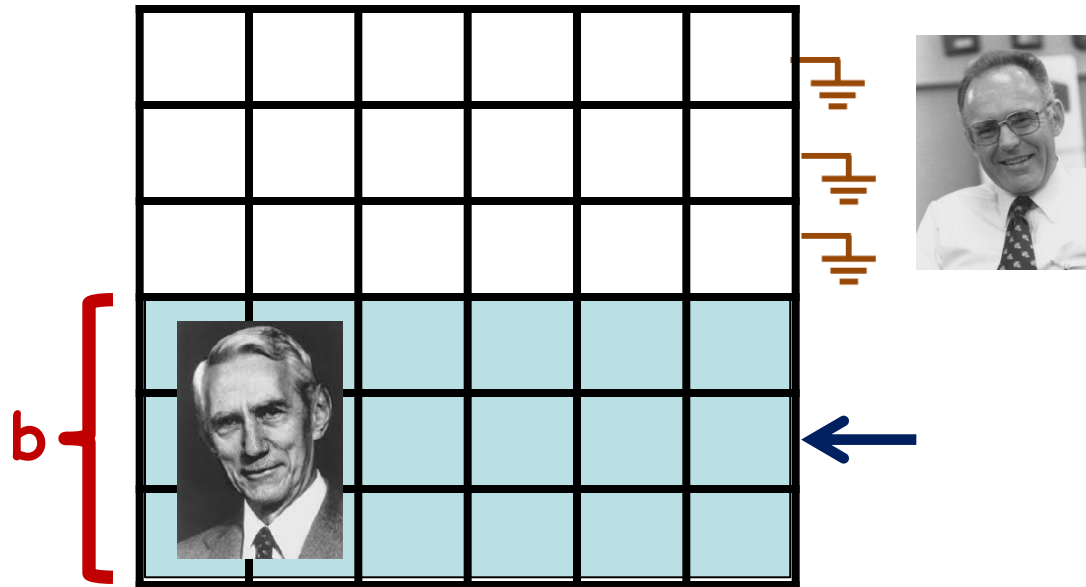


High read power



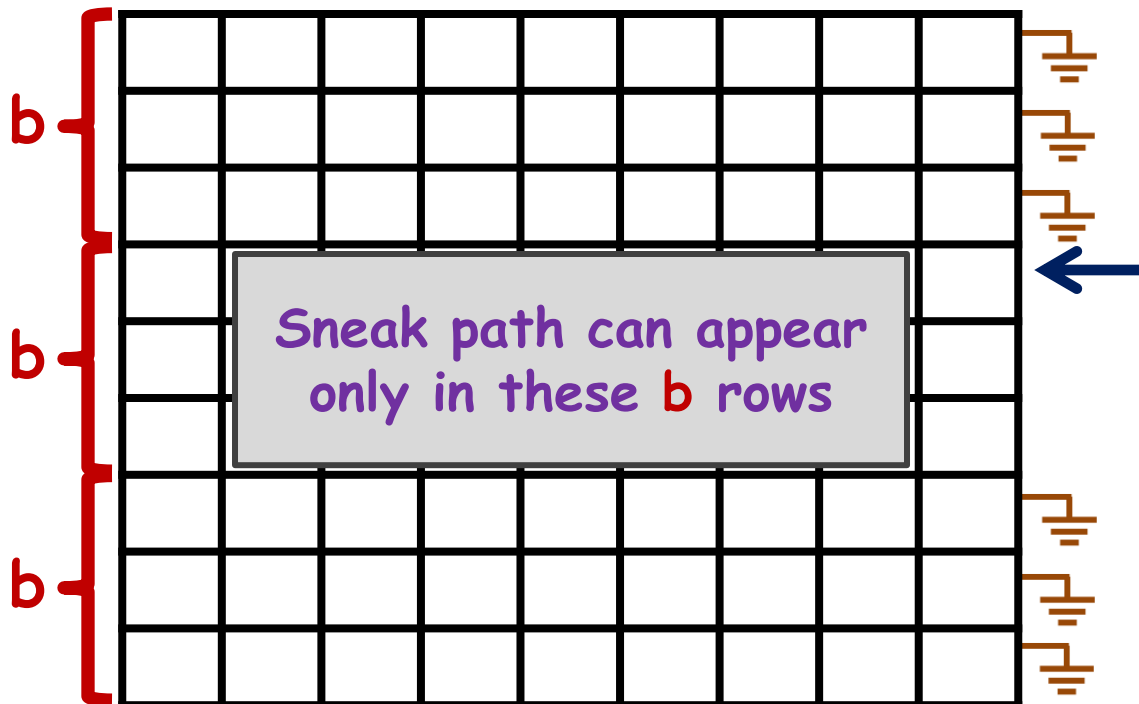
Poor capacity

A Mixed Solution



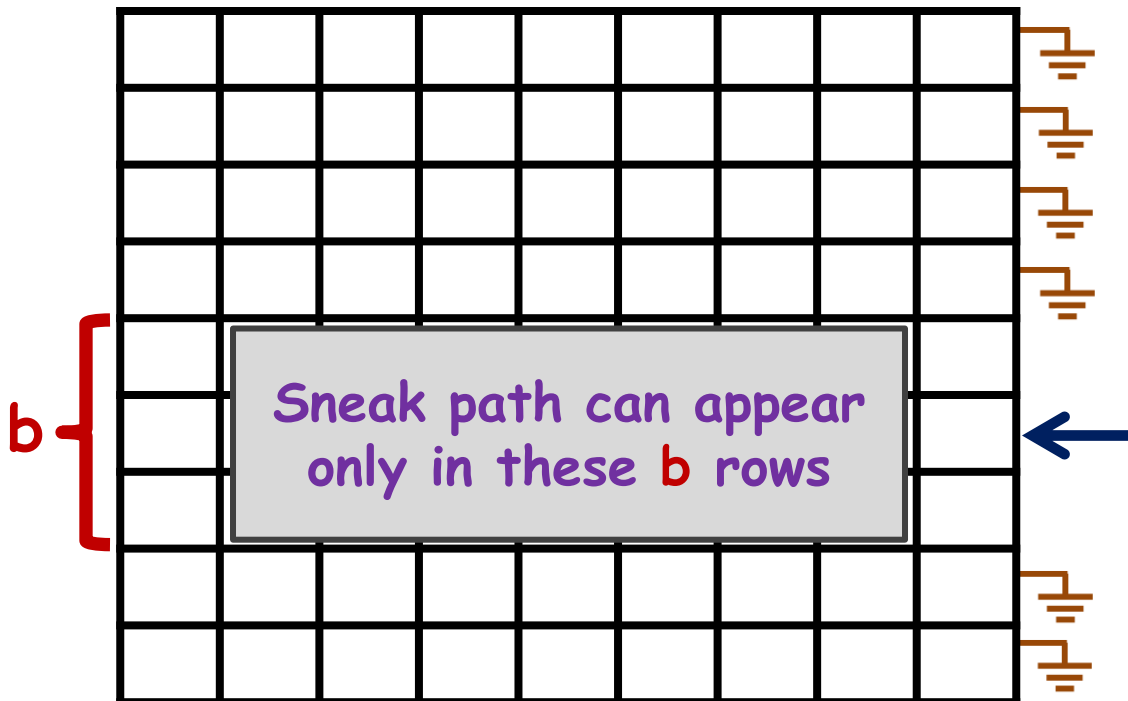
Grounding

- **Solution**: ground a smaller number of rows and combine with the coding solutions of the sneak path
- Two approaches to choose the grounded set
 1. Grounding **fixed subsets**



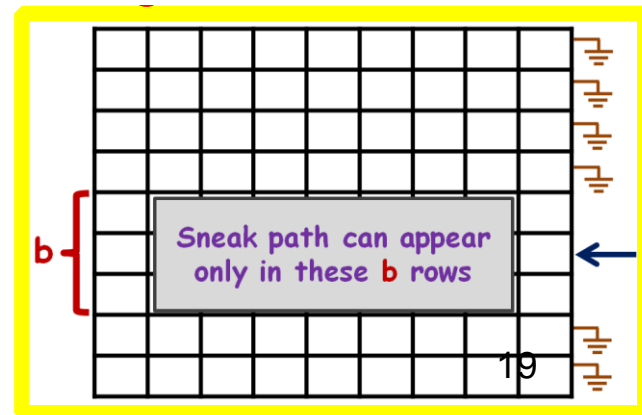
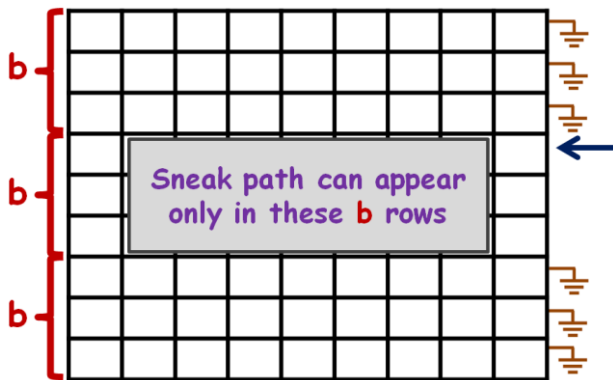
Grounding

- **Solution**: ground a smaller number of rows and combine with the coding solutions of the sneak path
- Two approaches to choose the grounded set
 1. Grounding **fixed subsets**
 2. Grounding **around** the read row



Comparison

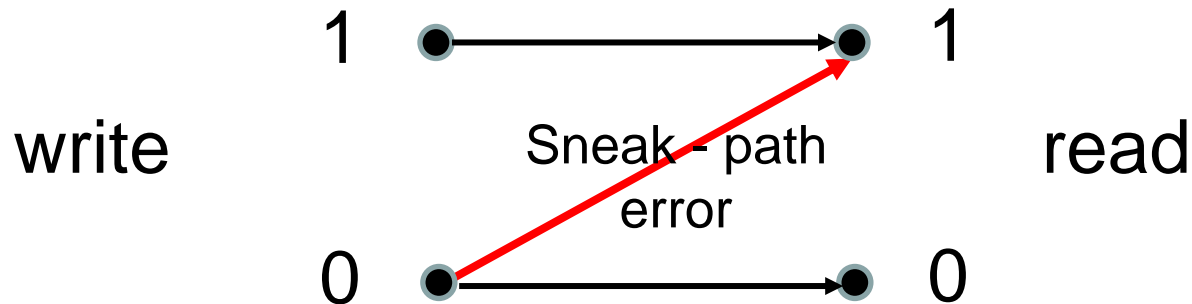
b	$\mathbb{C}_1(b) = \frac{\log(b+1)}{b}$	$\mathbb{C}_2(b) = \mathbb{C}_{\frac{b-1}{2}, \infty}$
2	0.792	-
3	0.667	0.694
4	0.580	-
5	0.517	0.551
6	0.468	-
7	0.423	0.465



Part 2

Sneak paths as a **random error source**

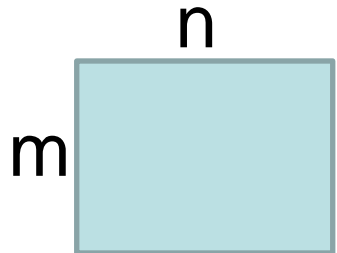
The Sneak-Path as Z Channel



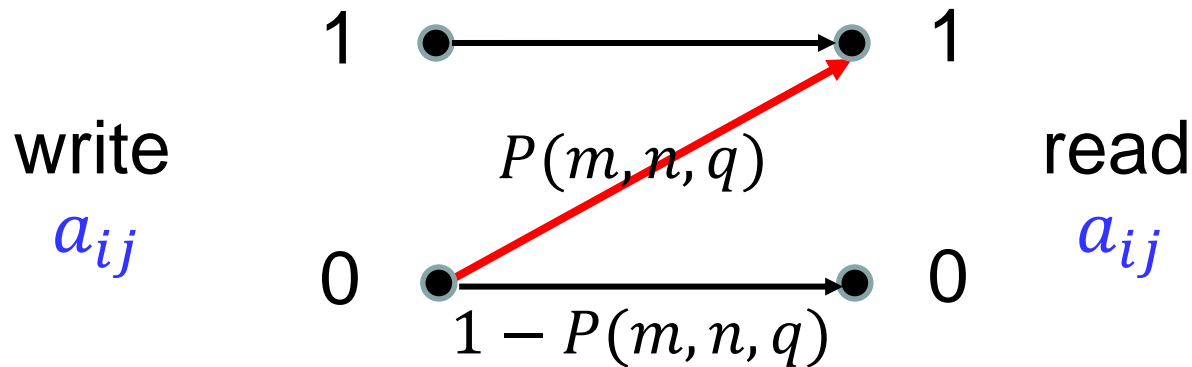
Errors are **deterministic** given array values

0	1	0	0	0
0	0	0	1	0
1	0	0	1	1
0	1	0	0	0
1	0	1	1	0

Sneak-Path Severity Factors

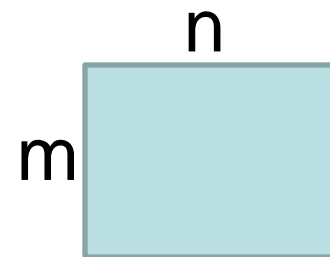
1. Array dimensions m, n 
 - Large array \rightarrow high vulnerability (more paths)
2. 0-1 bias q
 - More “1”s \rightarrow more sneak paths
 - (All “1”s \rightarrow no sneak paths)

The Sneak-Path **Z** Channel



Errors are **probabilistic** given array parameters:

1. Array dimensions **m,n**



2. 0-1 bias **q**

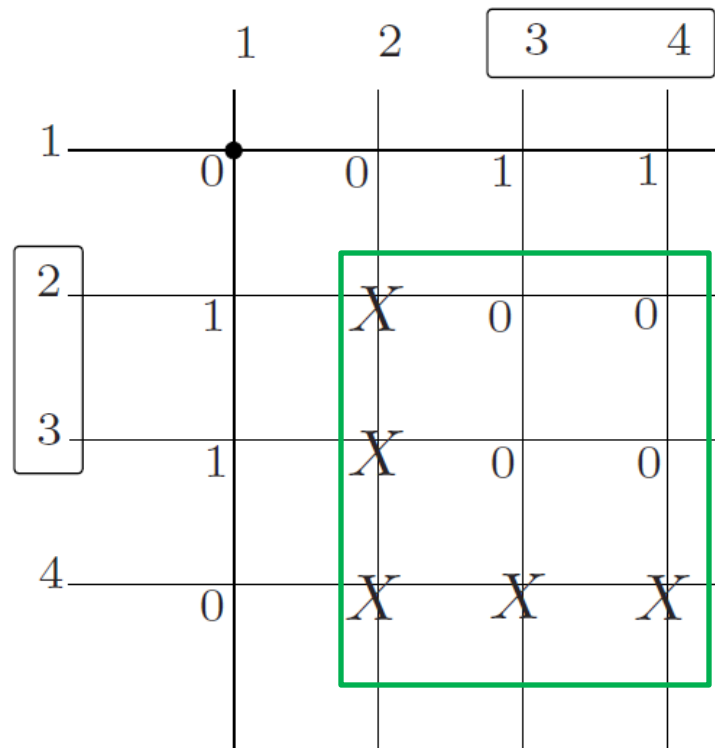
$$\Pr(a_{ij} = 1) = q \quad \Pr(a_{ij} = 0) = 1 - q$$

The Transition Probability

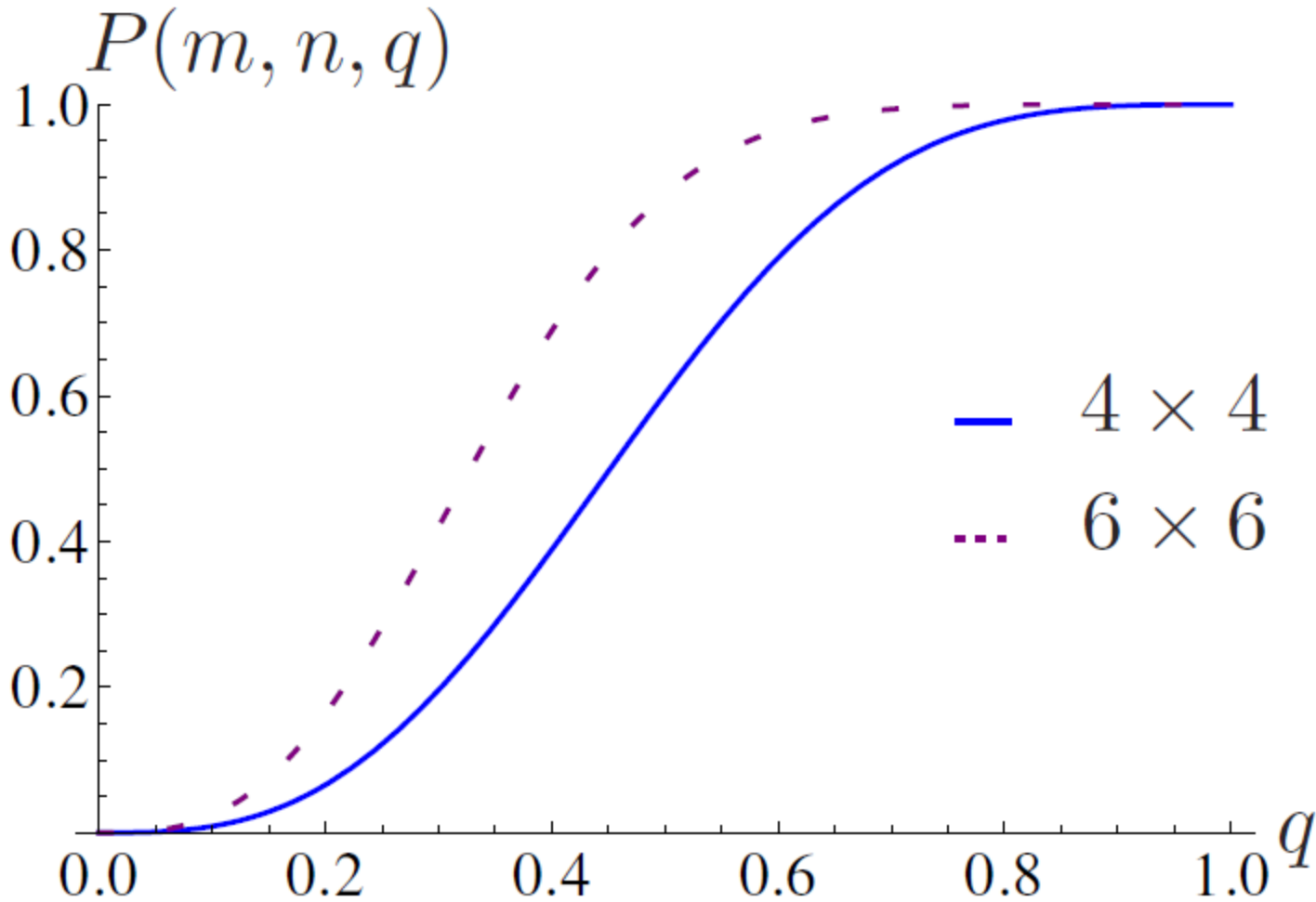
- Theorem:

$$P(m, n, q) = 1 - \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} \binom{m-1}{u} \binom{n-1}{v} q^{u+v} (1-q)^{m-1-u+n-1-v+uv}$$

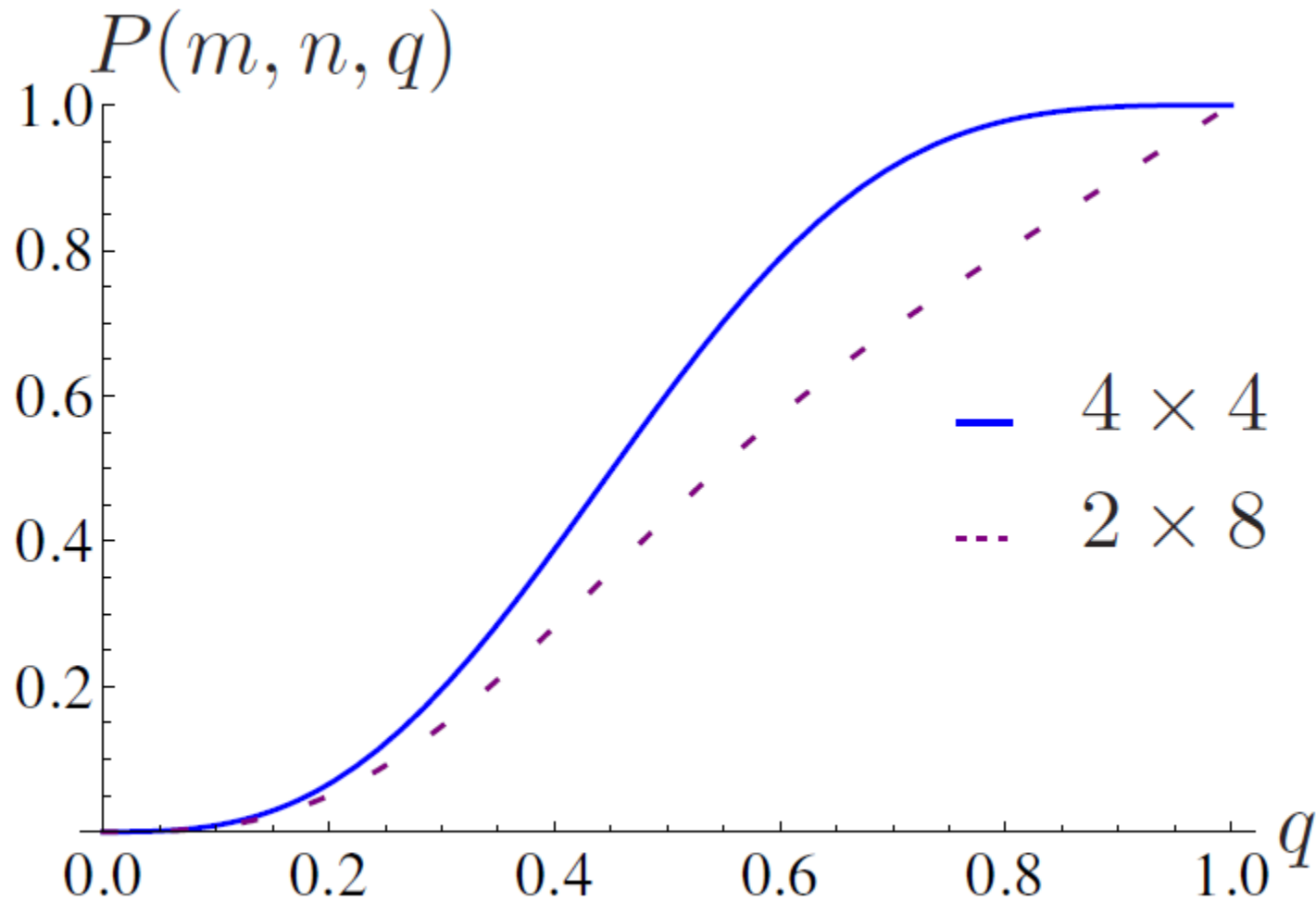
- Idea:



Transition Probability – Array Size



Transition Probability – Aspect Ratio



Joint-Probability Analysis

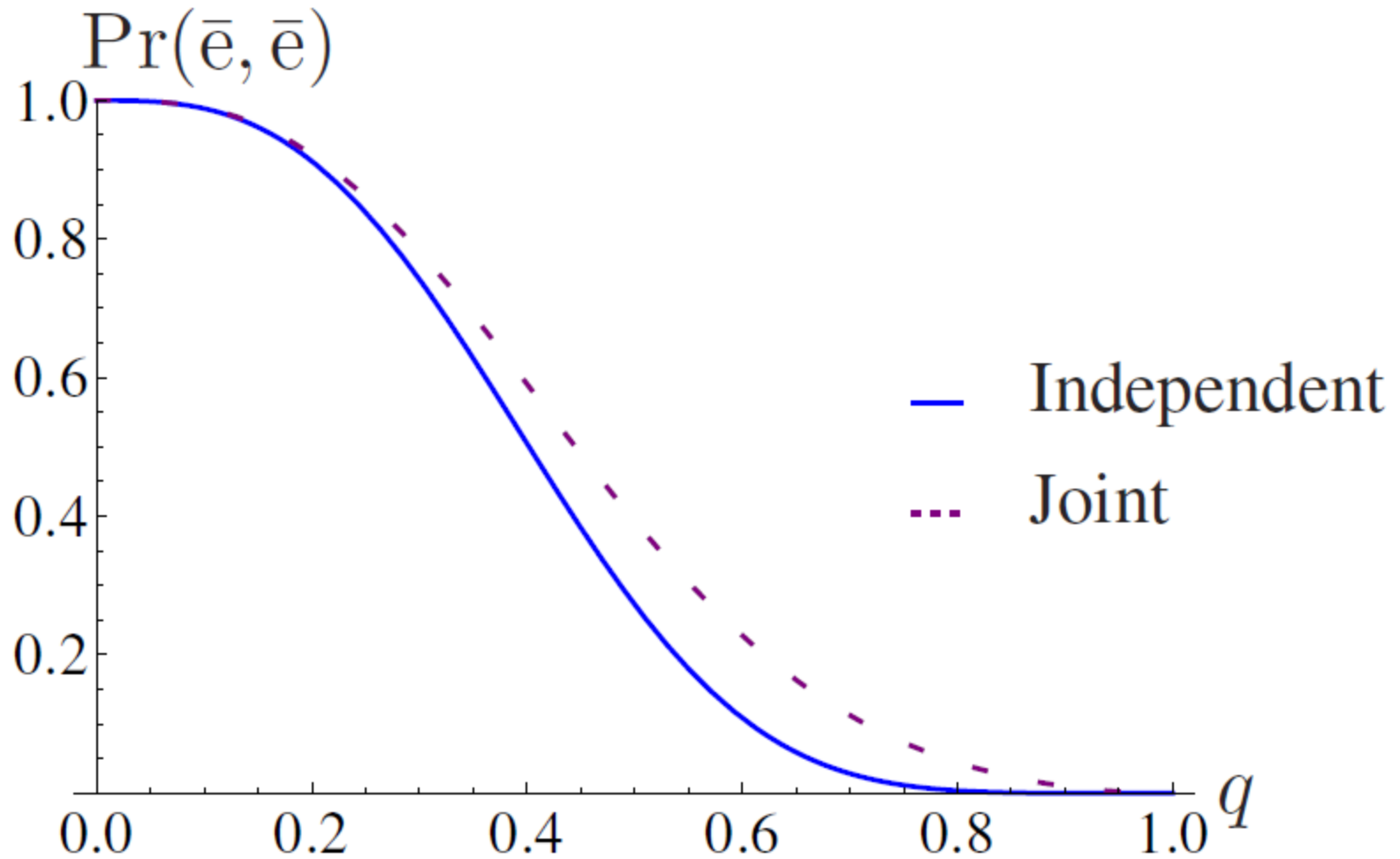
- Fact: sneak-path errors not independent within array
- Intuition: cells in same row/column share “1”s

- Theorem:

$$\Pr(\bar{e}_{i,j}, \bar{e}_{i',j}) =$$

$$\sum_{u=0}^{m-2} \sum_{v=0}^{n-1} \sum_{v'=0}^{n-1} \sum_{\sigma=0}^{n-1} \binom{m-2}{u} \binom{n-1}{v} \binom{v}{\sigma} \binom{n-1-v}{v'-\sigma} q^{u+v+v'} (1-q)^{m-2-u+n-1-v+n-1-v'+u(v+v'-\sigma)}$$

Joint-Probability: No Error



Joint-Probability: Error

