LECTURE 6: WRITE-ONCE MEMORY (WOM) – INTRODUCTION
The Erase Problem

0 → 1 easy
1 → 0 hard
Good transitions $0 \rightarrow 1$

What if all transitions were like that?

Somebody will have to pay something…
Solution: Re-write Codes

HOST

Write unrestricted

PHY WRITE

0 → 1 yes
1 → 0 no

Flash Media
Write-Once Memory (WOM)

Example: write 2 bits, 2 times. How many WOM bits are needed?

Trivial solution: 4 bits

<table>
<thead>
<tr>
<th>info</th>
<th>write 1</th>
<th>write 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>00 00</td>
<td>00 XX</td>
</tr>
<tr>
<td>01</td>
<td>00 01</td>
<td>01 XX</td>
</tr>
<tr>
<td>10</td>
<td>00 10</td>
<td>10 XX</td>
</tr>
<tr>
<td>11</td>
<td>00 11</td>
<td>11 XX</td>
</tr>
</tbody>
</table>
Write-Once Memory (WOM)

Example: write 2 bits, 2 times.
How many WOM bits are needed?

Clever solution: 3 bits

<table>
<thead>
<tr>
<th>info</th>
<th>write 1</th>
<th>write 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>000</td>
<td>111</td>
</tr>
<tr>
<td>01</td>
<td>001</td>
<td>110</td>
</tr>
<tr>
<td>10</td>
<td>010</td>
<td>101</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>011</td>
</tr>
</tbody>
</table>

Read only the last write!
WOM State Diagram

write 1

write 2 (partial)

All state transitions: 0 → 1 only
(n,k,t) WOM code:

Write k bits t times, on n cells
WOM = Speed + Long Life in Flash

IF
Time(Erase) ≫ Time(Write) \quad \text{Wear(Erase) ≫ Wear(Write)}
THEN
WOM speeds up by factor \( t \)
WOM extends life by factor \( t \)

0 \rightarrow 1 \text{ Write, } 1 \rightarrow 0 \text{ Erase}
Issues:
1. Redundancy
2. Read before write
WOM Code Definition

1) Decoding Function

\[ \psi \left( c_1, \ldots, c_n \right) = \left( y_1, \ldots, y_k \right) \]

\[ c_i \in \{0,1\} \]

2) Update Function

\[ \mu \left( c_1, \ldots, c_n ; y_1, \ldots, y_k \right) = \left( c'_1, \ldots, c'_n \right) \]

legal

current cell values new data new cell values
Requirements

• Consistency

\[ \psi [\mu (c_1, \ldots, c_n; \underline{y})] = \underline{y} \]

• Adherence

If \((c'_1, \ldots, c'_n) = \mu (c_1, \ldots, c_n; \underline{y})\), then \(\forall i, c'_i \geq c_i\)

• Completeness

\(\mu (c_1, \ldots, c_n; \underline{y})\) is defined and consistent for every \(\underline{y} \in \{0,1\}^k\)
Alternative Definition

\((n,k,t)\) WOM code \(\rightarrow \langle v \rangle^t / n\) WOM code

\(k = \log_2(v)\), but \(v\) not necessarily an integer power of 2.

Definition:

\(w(\langle v \rangle^t)\): number of cells in an optimal \(\langle v \rangle^t\) WOM code.
Lemma 1:

\[ w(\langle v_1 \cdot v_2 \rangle^t) \leq w(\langle v_1 \rangle^t) + w(\langle v_2 \rangle^t) \]

Proof:

Given \( y \in \{0, ..., v_1 v_2 - 1\} \), split to \( y = (y_1, y_2) \), where \( y_1 \in \{0, ..., v_1 - 1\} \), and \( y_2 \in \{0, ..., v_2 - 1\} \). Then concatenate

\[
\begin{array}{c|c}
  y_1 & y_2 \\
  \downarrow & \downarrow \\
  \langle v_1 \rangle^t & \langle v_2 \rangle^t \\
\end{array}
\]
Reduction on Writes

Lemma 2:

\[ w(\langle v \rangle^{t_1+t_2}) \leq w(\langle v \rangle^{t_1}) + w(\langle v \rangle^{t_2}) \]

Proof:

\[ c_1 \]

\[ c_2 \]

\[ y_1 \]
\[ y_2 \]
\[ y_{t_1} \]

\[ y_{t_1+1} \]
\[ y_{t_1+2} \]
\[ y_{t_1+t_2} \]
Write-Count Issue

Option 1: $y = \psi(c_1)$, if $t_1$ writes or less

Option 2: $y = \psi(0)$, if $t_1 + 1$ writes

Must know the number of writes!
Avoiding Write Counters

**Lemma 2:**

\[ w(\langle \nu \rangle^{t_1+t_2}) \leq w(\langle \nu \rangle^{t_1}) + w(\langle \nu \rangle^{t_2}) \]

**Proof:**

\[ \psi(c) \triangleq \psi(c_1) - \psi(c_2) \]

Move to \( c_2 \) only with non-zero!
Reduction on Input and Writes

**Theorem:**

\[ w(\langle 2^k \rangle^t) \leq kt \]

**Proof:**

\[ w(\langle 2^k \rangle^t) \leq k \cdot w(\langle 2 \rangle^t) \leq kt \cdot w(\langle 2 \rangle^1) = kt \]

Lemma 1 \[
\uparrow
\]

Lemma 2

\[ = 1 \]