1. Variable-Rate Rewrite

You are given \( n = 2 \) cells with \( q = 4 \) levels \( \{0, 1, 2, 3\} \) whose legal transitions are as usual:
\( 0 \rightarrow \{0, 1, 2, 3\}, 1 \rightarrow \{1, 2, 3\}, 2 \rightarrow \{2, 3\} \) and \( 3 \rightarrow \{3\} \).

a) Design a code (update and decoding functions) for \( t = 2 \) writes with maximal \( v \) (same \( v \) for both writes).

b) Can you find a code with \( t \neq 2 \) and some \( v \) that has a higher sum-rate than the code in (a)? Show the code with maximal sum-rate.

c) Design a code for \( t = 2 \) writes as follows: \( v_1 \) possible data values in the first write and \( v_2 \) possible data values in the second (\( v_1 \) and \( v_2 \) are allowed to be different). Find a code with the maximal sum-rate. Note: when \( v_1 \neq v_2 \) you can assume that upon read request the device knows the current write generation.

d) Compare the sum-rates of your codes in (a) and (c) with the appropriate capacity upper bound.

2. Huffman and LZ Compression

In this problem you will exercise with Huffman and LZ compression, and their comparison.

a) Calculate a Huffman code for the distribution \( (0.4, 0.3, 0.16, 0.08, 0.06) \).

b) Find a real-world distribution on 5 elements (e.g., days of lecture in EE courses) and calculate a Huffman code for it.

c) Draw a sequence of 100 instances from the distribution in (a). Represent each element as a byte (8 bits) and run LZ77 compression on it. Show the compressed sequence and compare the compression ratio to Huffman coding.

d) Find a way to improve the LZ compression ratio from (c).