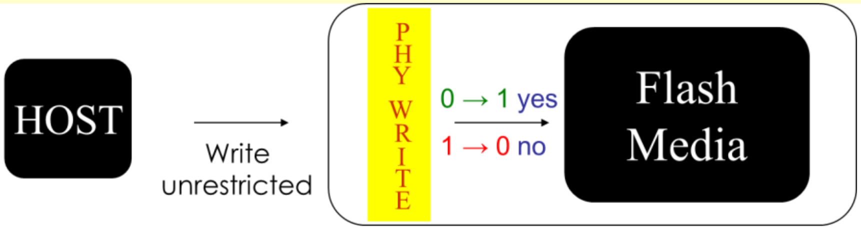
Information in Storage Devices

049063 – EE Department, Technion

LECTURE 7: WRITE-ONCE MEMORY (WOM) – CONSTRUCTIONS AND BOUNDS

WOM Quiz



HOST

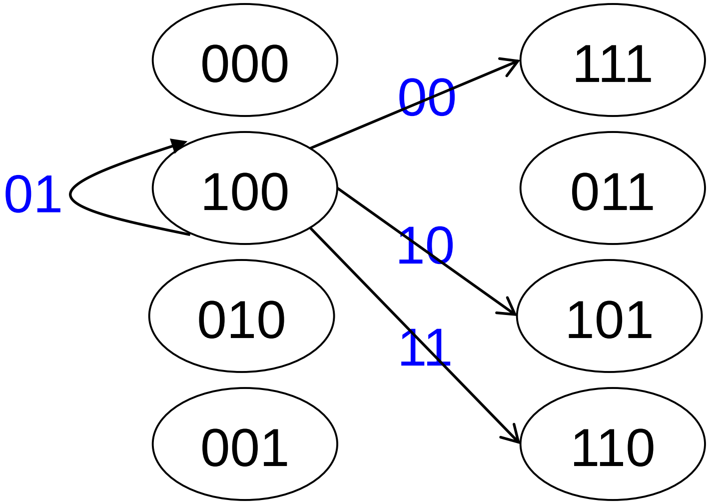
Write unrestricted

PHY WRITE

0 → 1 yes

1 → 0 no

Flash Media



$$\psi(c_1,\ldots,c_n)=(y_1,\ldots,y_k)$$

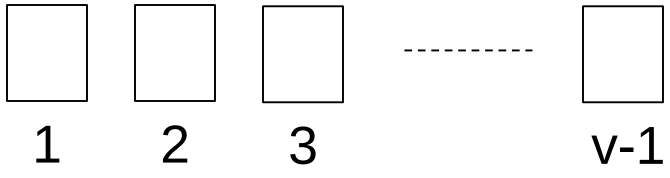
$$w(\langle 2^k\rangle^t)\leq kt$$

$$w(\langle v\rangle^{t_1+t_2})\leq w(\langle v\rangle^{t_1})+w(\langle v\rangle^{t_2})$$

$$\mu(c_1,\ldots,c_n\,;\,y_1,\ldots,y_k)=(c'_1,\ldots,c'_n)$$

The Linear WOM Code

v, n=v-1, t=v/4



Decoding function:

$$x = \sum_{i=1}^{v-1} i c_i \bmod v$$

Update function:

Current info: x
New info: y

Change c_is 0→1 such that $y = \sum_{i=1}^{v-1} i {c_i}' \bmod v$

↑
Minimally!

Linear WOM Code – Example

v=6, n=5

Decoding:

1	0	0	1	0
1	2	3	4	5

$$x = (1 + 4) \bmod 6 = 5$$

Update:

New info: $y = 0$

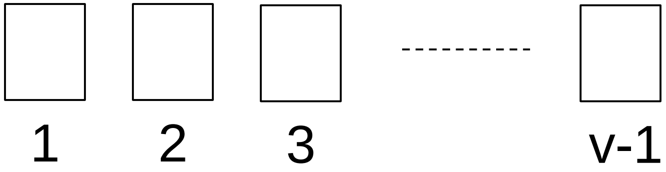
Need +1, but cell 1 already “1”

Update +7 instead

1	1	0	1	1
1	2	3	4	5

$$(1 + 2 + 4 + 5) \bmod 6 = 0 = y$$

Linear WOM Code - Update

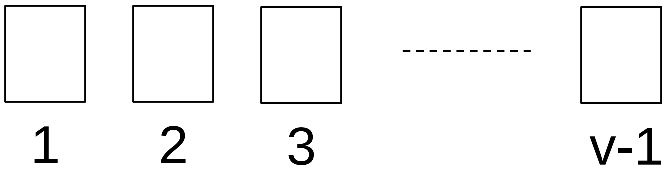


Add (y-x)mod v to the array by “minimal” 0⟶1 changes.
Define z=(y-x)mod v

Case 1: $c_z = 0$

$c_z: 0 \rightarrow 1$

Linear WOM Code - Update



Add (y-x)mod v to the array by “minimal” 0→1 changes.
Define z=(y-x)mod v

Case 2: $c_z = 1$

Define S to be the set of zero bits: $S = \{i: c_i = 0\}$

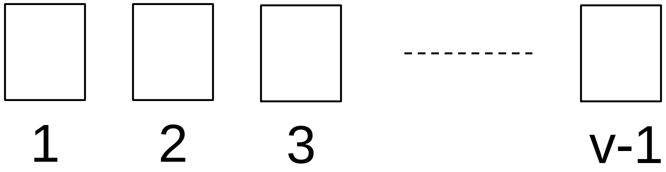
Assumption: $|S| \geq v/2$

Define T as the set $T = \{(z - i) \bmod v \mid i \in S\}$

Fact: $|\mathrm{T}| = |S| \geq v/2$

Linear WOM Code – 2-bit Update

z=(y-x)mod v $S = \{i : c_i = 0\}$ $T = \{(z - i) \bmod v \mid i \in S\}$



$$\left.\begin{array}{ll} \text{Fact 1: z not in S} & \text{Otherwise Case 1} \\ \text{Fact 2: z not in T} & \text{There is no } c_0 \\ \text{Fact 3: z not 0} & \text{That would mean y=x} \end{array}\right\} |T \cup S| < v - 1$$

If $|S|, |T| \geq \frac{v}{2}$, $|T \cup S| < v - 1$ then $\boxed{|T \cap S| \geq 2}$

Linear WOM Code – 2-bit Update

z=(y-x)mod v $S = \{i: c_i = 0\}$ $T = \{(z - i) \bmod v \mid i \in S\}$

$$\boxed{|T \cap S| \geq 2}$$

1) Choose i' from the intersection $T \cap S$
2) i'=(z-i'') mod v, for some i'' in S
3) Both i',i'' in S, and i'+i''=z mod v

Add z to the array by $c_{i'}: 0 \to 1$ $c_{i''}: 0 \to 1$

≤2 0→1 changes for every write

Guaranteed v/4 writes!

Getting More than n/4 Writes

What to do with the remaining v/2 cells still at zero?

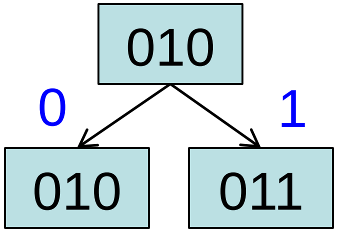
Divide the block to n/3 triples:

110	011	100	111	010

Ex. n=15

Rules:

1) If 0 “1”s: add arbitrary “1”
2) If 2 “1”s: add a “1” to get 111
3) Now re-use all **triples with a single “1”**



Single “1” means 0

Two “1”s means 1

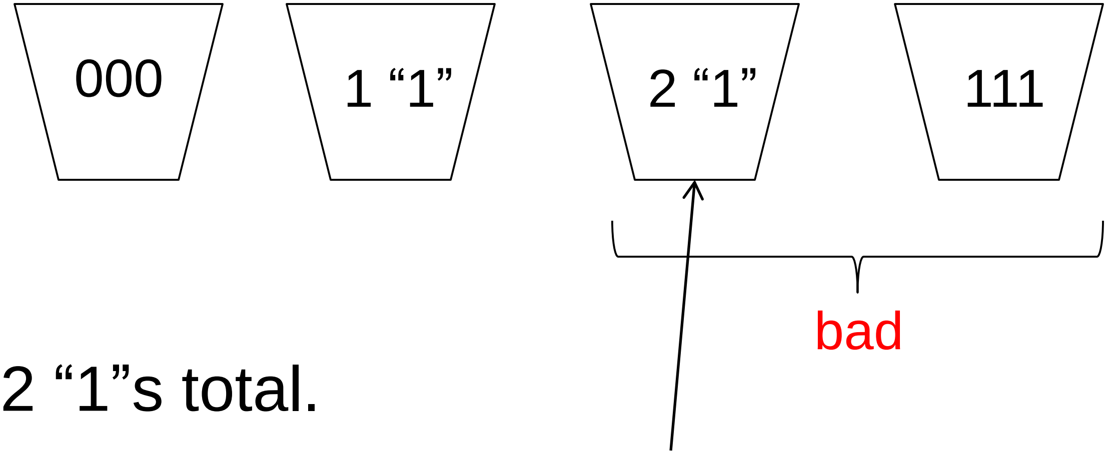
Getting More Than n/4 Writes

Ex. n=15

After v/4 writes	110	011	100	111	000
Prepare next write	111	111	100	111	010
After next write	111	111	101	111	010
	x	x	1	x	0

How Many More Writes?

How many additional bits from triples?



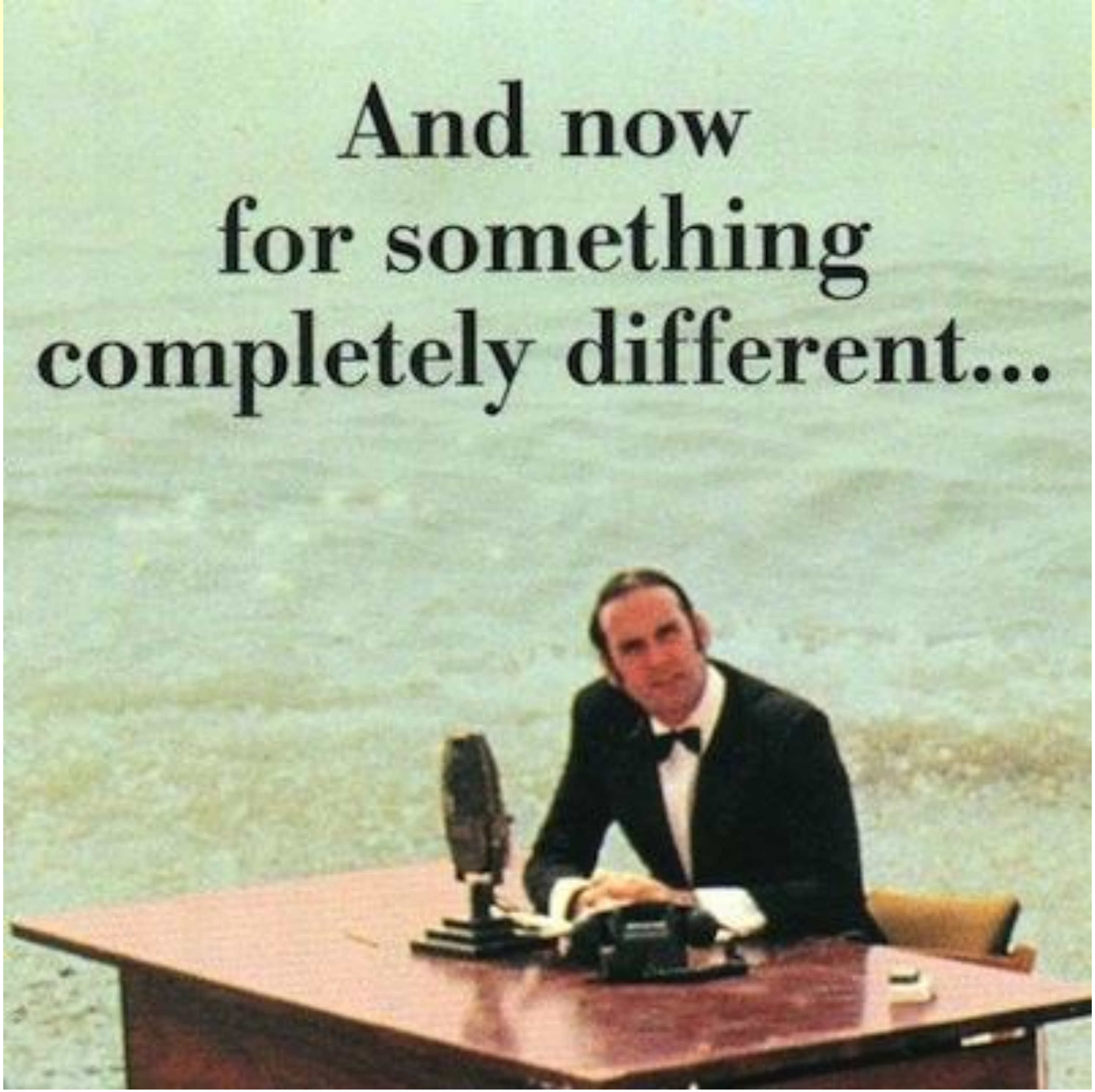
n/3 triples, n/2 “1”s total.

Worst case: n/2 “1”s in n/4 triples **with 2 “1”s**

$$\# \text{ additional bits} \geq n/3 - n/4 = n/12$$

Recursion: $t(n) = n/4 + t(n/12)$

\# writes total: $t(n) = 3n/11 + O(1)$ → Recursion improved t by factor 12/11


And now
for something
completely different...

Lower Bound on $w(\langle v\rangle^{2})$

v, t=2, n≥?

The first write needs to leave at least $\log_2(v)$ zero bits

Define $m = \log_2(v)$ and $h = n - m$ bits for first write

Condition on h from first write:

$$\sum_{i=0}^{h}\binom{n}{i} = \sum_{i=0}^{h}\binom{m+h}{i} \geq v$$

$$w(\langle v\rangle^2) \geq m + \min\left\{h: \sum_{i=0}^{h}\binom{m+h}{i} \geq v\right\}, m = \log_2(v)$$

Lower Bound on $w(\langle v\rangle^{t})$

v, t, n ≥?

Define $$\delta(v,m)=\min\left\{h:\sum_{i=0}^{h}\binom{m+h}{i}\geq v\right\}$$

Theorem: $$w(\langle v\rangle^{t})\geq z(v,t)$$

where $$z(v,0)=0$$

$$z(v,t+1)=z(v,t)+\delta\big(v,z(v,t)\big) \qquad t\geq 0$$

Proof: by induction

For first write | Zeros for writes 2,…,t+1

$\delta\big(v,z(v,t)\big)$ | $z(v,t)$

Explicit Lower Bound on $w(\langle v\rangle^{t})$

$v=2^k$, t, $n \geq ?$

Corollary:

$$w(\langle 2^k\rangle^{t}) \geq k+t-1$$

Proof:

$$z(2^k,1) = \delta(2^k,0) = k$$

$$z(2^k,t+1) \geq z(2^k,t) + \boxed{1}$$

Last write: k bits

Each earlier write, at least 1 additional bit

Explicit Lower Bound on $w(\langle v \rangle^{\mathbf{2}})$

v, t=2, n ≥?

Theorem: $w(\langle v \rangle^2) \geq 1.293 \log_2(v)$

Proof:

$$\sum_{i=0}^{n-\log_2(v)} \binom{n}{i} \geq v$$

The asymptotic wand

$$2^{nH\left(\frac{n-\log_2(v)}{n}\right)} \geq \ v$$

The binary entropy function

Then solve $H\left(1 - \frac{\log_2(v)}{n}\right) \geq \frac{\log_2(v)}{n}$ and get $n \geq 1.293 \log_2(v)$

WOM Capacity

Definition: WOM Capacity

Define the **t-write WOM capacity C_t** as the total number of bits that can be written in all t writes, normalized by the number of cells n.

t=2

$$w(\langle v\rangle^2) \geq 1.293\log_2(v)$$

$$\downarrow$$

t

$$C_2 \leq \frac{2\log_2(v)}{1.293\log_2(v)} = 1.54$$

binary cells

Are you sure?

Variable-Rate WOM

t=2

We can write v_1 in the first write and v_2 in the second write. Will it help?

Bits for first write

Bits for second write

pn

$(1-p)n$

$$v_2 = \quad 2^{(1-p)n}$$

$$v_1 = \sum_{i=0}^{pn} \binom{n}{i} \quad \approx 2^{nH(p)}$$

Maximal Sum-Rate

t=2

Total number of bits:

$$\log_2(v_1) + \log_2(v_2) = nH(p) + (1-p)n$$

Maximize over p:

$$\overline{p} = \mathrm{argmax}_p[H(p) + (1-p)] = \frac{1}{3}$$

$$C_2 \leq H(\overline{p}) + (1-\overline{p}) = 1.58 > 1.54$$

0.918 0.666